# 1st Order ODE Practice Questions

**Problem 1:**
Solve the following initial value problem:

$$y' = (2x+3)(y^2-4), \quad y(0) = -1$$

**Solution:**
The solution to the initial value problem is:

$$y = \frac{2 - 6e^{4x^2+12x}}{1 + 3e^{4x^2+12x}}$$

**Problem 2:**
Solve the following initial value problem:

$$x\frac{dy}{dx} - y = x^2, \quad y(1) = 2$$

**Solution:**
The solution to the initial value problem is:

$$y = x^2 + x$$

**Problem 3:**
Solve the following first order differential equation:

$$\frac{dy}{dx} - \sin x \cdot y = \sin x$$

**Step 1: Rewrite in standard form**
The equation is already in standard form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x) = -\sin x$ and $Q(x) = \sin x$.
**Step 2: Find the integrating factor**
The integrating factor $\mu(x)$ is given by:

$$\mu(x) = e^{\int P(x)\,dx} = e^{\int -\sin x\,dx} = e^{\cos x}$$

**Step 3: Multiply through by the integrating factor**
Multiply both sides of the equation by the integrating factor $e^{\cos x}$:

$$e^{\cos x}\frac{dy}{dx} - e^{\cos x}\sin x \cdot y = e^{\cos x}\sin x$$

The left-hand side can be expressed as the derivative of $ye^{\cos x}$:

$$\frac{d}{dx}\left(ye^{\cos x}\right) = e^{\cos x}\sin x$$

**Step 4: Integrate both sides**

Integrate both sides with respect to $x$:

$$ye^{\cos x} = \int e^{\cos x}\sin x\,dx + C$$

**Step 5: Solve for $y$**

Now, solve for $y$:

$$y = e^{-\cos x}\left(\int e^{\cos x}\sin x\,dx + C\right)$$

**Step 6: Evaluate the integral**

To evaluate the integral $\int e^{\cos x}\sin x\,dx$, we can use substitution. Let:

$$u = \cos x \quad \Rightarrow \quad du = -\sin x\,dx \quad \Rightarrow \quad -du = \sin x\,dx$$

Thus, the integral becomes:

$$\int e^{u}(-du) = -e^{u} + C = -e^{\cos x} + C$$

**Final Answer**

Substituting back into the equation for $y$, we have:

$$y = e^{-\cos x}\left(-e^{\cos x} + C\right)$$

Simplifying gives:

$$y = -1 + Ce^{-\cos x}$$

Thus, the final solution to the differential equation is:

$$y = Ce^{-\cos x} - 1$$

**Problem 4:** Solve the differential equation

$$2y' + y = 3x^2.$$

**Solution:**

$$y = 3x^2 - 12x + 24 + Ce^{-\frac{1}{2}x}.$$

**Problem 5:**

The differential equation is given by:

$$y' - y = 1 + 3\sin x,$$

with the initial condition:

$$y(0) = y_0.$$

Find the value of $y_0$ for which the solution of the initial value problem remains finite as $t \to \infty$.

**Answer:**

$$y_0 = \frac{-5}{2}$$

**Problem 6:** Solve the first-order ordinary differential equation (ODE) given by:

$$y' = e^{2x} + y - 1.$$

**Answer:**

$$y = e^{2x} + 1 + ce^{x}$$

**Problem 6.1:** The given differential equation in Problem 6 is not exact; however, it can become an exact equation if multiplied by an appropriate integrating factor. Find this integrating factor and solve exact equation, then compare results you obtained in Problem 6 and Problem 6.1.

**Problem 7:** Solve the following differential equation.

$$(2xy^2 + 2y) + (2x^2y + 2x)y' = 0.$$

**Answer:** The solution to the equation is

$$x^2y^2 + 2xy = c,$$

where $c$ is a constant.

**Problem 8:** Solve the following differential equation:

$$xy' = \sqrt{1 - y^2}.$$

**Answer:** The solution to the equation is

$$y = \sin(\ln|x| + c),$$

where $x \neq 0$ and $|y| < 1$.

**Problem 9:** Solve the following differential equation:

$$y' = \frac{y - 4x}{x - y}.$$

**Hint:** This differential equation can be classified as separable if an appropriate change of variables is made.

**Answer:**

$$(y - 2x)(y + 2x)^3 = exp(-4c)$$

**Problem 10:** Solve the following differential equation:

$$t^2y' + 2ty - y^3 = 0.$$

**Hint:** This differential equation is nonlinear; however, an appropriate change of variables can transform it into a linear one.

**Answer:** The solution to the equation is

$$y = \left(\frac{5t}{2 + 5ct^5}\right)^{\frac{1}{2}},$$

where $c$ is a constant.

**Problem 11:** Solve the following differential equation.

$$dx + (\frac{x}{y} - siny)dy = 0$$

**Answer:**

$$\nu(y) = y; xy + ycosy - siny = c$$